Il rinforzo degli archi in muratura

Experimental and numerical analysis of masonry arches reinforced with FRP materials

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Outline of the presentation

- Motivations
- Governing equations
- Numerical procedures
  - Stress formulation
  - Displacement formulation FEM
- Post-computation of the stresses
- Numerical applications
- Experimental campaign
- Comparison between numerical and experimental results
- Conclusions & future developments
Costruzioni in muratura

Il patrimonio edilizio e monumentale di un Paese sintetizza la storia, le tradizioni e la civiltà del suo popolo. Nei Paesi con più antiche tradizioni, tale patrimonio è costituito quasi esclusivamente da strutture in muratura. (Romano-Sacco 1983)

Il collasso di strutture murarie, pannelli, archi, volte, è una delle maggiori cause di perdita di vite umane durante gli eventi sismici.
Motivations

Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Existing Structures

CNR DT/200
Objective

analysis of reinforced masonry arches

- Simple strategies for the engineers;
- Accurate analysis to validate the simple strategies.
Masonry arches

CAUSES OF FAILURE
• Deficiency of design;
• Decay of used materials;
• Use destination variation of the structure;
• Foundation settlements.

REMEDIES
• …
• use of FRP
• …

COLLAPSE MECHANISM

FRP
• very light, excellent mechanical properties
• very simple and fast to applying
• reduced of the cost
Masonry arc subjected to vertical and horizontal loading
Hinges in the masonry arc
Kinematics of the arc at the collapse
Kinematics of the arc at the collapse

The reinforcement at the extrados inhibits the opening of the hinge

The reinforcement at the intrados inhibits the opening of the hinge
Kinematics of the arc at the collapse
(shear collapse in a section)

The reinforcement at the extrados inhibits the opening of the hinge

The reinforcement at the intrados inhibits the opening of the hinge

The reinforcement does not induce any positive effect
3 BASIS OF DESIGN FOR FRP STRENGTHENING
(1)P The subject of this chapter regards FRP strengthening of existing … masonry structures ….

3.1 BASIC REQUIREMENTS
……
(3)P If FRP strengthening concerns structures of historical and monumental interest, a critical evaluation of the strengthening technique is required with respect to the standards for preservation and restoration. The actual effectiveness of the strengthening technique shall be objectively proven, and the adopted solution shall guarantee compatibility, durability, and reversibility.

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5.1.2 Strengthening of historical and monumental buildings
(1)P Strengthening of historical and monumental buildings shall be justified only when inevitable; the adopted strengthening technique shall be in compliance with the theory of restoration.
Principi (Carta di Venezia)

• non mutare il carattere stesso della struttura
• essere reversibile, soprattutto quando non è noto il comportamento a lungo termine
• essere distinguibile dall’originale composizione architettonica, portando con se la data dell’intervento stesso

Applicazione: miglioramento sismico delle volte del Palazzo Comunale di Assisi
Collapse mechanism of reinforced arch

- Tensile failure of the masonry
- Crushing of masonry in compression;
- Decohesion of FRP;
- Failure of the FRP.
Arch governing equations

Equilibrium equation \( \Delta c + p^* + \lambda q^* = 0 \)

\[
\Delta = \begin{bmatrix}
\frac{d}{ds} & 0 & \frac{1}{R} \\
0 & \frac{d}{ds} & -1 \\
-\frac{1}{R} & 0 & \frac{d}{ds}
\end{bmatrix}
\]

\[
c = \begin{bmatrix} N \\ M \\ T \end{bmatrix}, \quad p^* = \begin{bmatrix} p_s^* \\ 0 \end{bmatrix}, \quad q^* = \begin{bmatrix} q_s^* \\ 0 \end{bmatrix}
\]

Deformation vector

\[
d = \begin{bmatrix} \varepsilon_0 \\ \chi \end{bmatrix}
\]
Constitutive equations

**MASONRY**

\[
\sigma_M = E_M \left( \varepsilon - \delta - p \right)
\]

\[
\tau_M = G_M \left( \gamma - g \right)
\]

if \( \varepsilon > 0 \)

\[
\begin{cases}
\delta = \varepsilon \\
p = 0 \\
g = \gamma
\end{cases}
\]

if \( \varepsilon_y < \varepsilon \leq 0 \)

\[
\begin{cases}
\delta = 0 \\
p = -h_M \frac{\sigma_y}{\varepsilon_y^2} \varepsilon^2 \\
g = 0
\end{cases}
\]

if \( \varepsilon \leq \varepsilon_y \)

\[
\begin{cases}
\delta = 0 \\
p = \varepsilon - \varepsilon_y \\
g = 0
\end{cases}
\]

\( \varepsilon, \gamma \) total strain

\( \delta, g \) fracture strain

\( p \) plastic strain

\[
h_M = \begin{cases} 
0 & \text{for } M1 \\
1 & \text{for } M2 
\end{cases}
\]
Constitutive equations

MASONRY model $\mathcal{M}1$

\[
\begin{align*}
\sigma_M &= 0 \quad \text{if } \varepsilon \geq 0 \\
\tau_M &= 0 \\
\sigma_M &= E_M \varepsilon \quad \text{if } \varepsilon_y < \varepsilon < 0 \\
\tau_M &= G_M \gamma \\
\sigma_M &= E_M \varepsilon_y \quad \text{if } \varepsilon \leq \varepsilon_y \\
\tau_M &= G_M \gamma
\end{align*}
\]

Complementary energy

\[
\psi(\sigma_M, \tau_M) = \frac{1}{2E_M} \sigma_M^2 + I_\Sigma(\sigma) + \frac{1}{2G_M} \tau_M^2 (1 - h(\sigma_M))
\]

Indicator function of $\Sigma = ]\sigma_y, 0[$

Heaviside’s function

Ricamato-Marfia-Sacco, 2007
Constitutive equations

FRP: linear elastic

\[ \sigma_R = E_R \varepsilon \]

Complementary energy

\[ \psi (\sigma_M, \tau_M) = \frac{1}{2E_R} \sigma_R^2 \]

Control on the strain value:

\[ \varepsilon_{yR}^- \leq \varepsilon \leq \varepsilon_{yR}^+ \]

Perfect adhesion between the FRP and the masonry!!!!!
Cross section

Stress resultants

\[ \mathbf{c} = \begin{bmatrix} \hat{c} \\ T \end{bmatrix} = \begin{bmatrix} N \\ M \\ T \end{bmatrix} = \begin{bmatrix} \int_{A_p} \sigma_y \, dA + \int_{A_p} \left( \sigma_0 + y^* \sigma_1 \right) \, dA + N_R = \sigma_y A_p + A_e \sigma_0 + S_e \sigma_1 + N_R \\ \int_{A_p} y^* \sigma_y \, dA + \int_{A_e} y^* \left( \sigma_0 + y^* \sigma_1 \right) \, dA + M_R = \sigma_y S_p + S_e \sigma_0 + I_e \sigma_1 + M_R \\ \int_{A_e \cup A_p} \tau_M \, dA = \tau_M A_S \end{bmatrix} \]

\( A_p \) plastic part of the cross section

\( A_e \) elastic part of the cross section

\( A_{nt} \) no-tension part of the cross section

\( \chi \geq 0 \)
\[ \chi \geq 0 \]

\[ A_p \, y_1 \leq y \leq y_3 \]
\[ A_e \, y_3 < y \leq y_2 \]

\[ y_1 = \frac{-h}{2} \]
\[ y_2 = \min \left\{ \frac{h}{2}, \max \left\{ -\frac{h}{2}, y^*_n \right\} \right\} \]
\[ y_3 = \min \left\{ \frac{h}{2}, \max \left\{ -\frac{h}{2}, y^*_m \right\} \right\} \]

\[ \chi < 0 \]

\[ A_p \, y_3 \leq y \leq y_2 \]
\[ A_e \, y_1 \leq y < y_3 \]

\[ y_1 = \min \left\{ \frac{h}{2}, \max \left\{ -\frac{h}{2}, y^*_n \right\} \right\} \]
\[ y_2 = \frac{h}{2} \]
\[ y_3 = \min \left\{ \frac{h}{2}, \max \left\{ -\frac{h}{2}, y^*_m \right\} \right\} \]

\[ 0 = \varepsilon_0 + y^*_n \chi \quad \implies \quad y^*_n = -\frac{\varepsilon_0}{\chi} \]
\[ \varepsilon_y = \varepsilon_0 + y^*_m \chi \quad \implies \quad y^*_m = \frac{\varepsilon_y - \varepsilon_0}{\chi} \]
Stress formulation

\[ \Psi(\sigma, \tau) = \int_{V_M} \psi_M(\sigma_M, \tau_M) \, dV + \int_{V_R} \psi_R(\sigma_R) \, dV \]

Masonry energy

\[
\int_{V_M} \psi_M(\sigma_M, \tau_M) \, dV = \frac{\theta_f}{\theta} \left[ \frac{1}{2E_M} \int_{A_p} \sigma_M^2 \, dA + \frac{1}{2E_M} \int_{A_v} \sigma_M^2 \, dA \right] + \frac{1}{2G_M} \int_{A_v \cup A_p} \tau_M^2 \, dA R \, d\theta
\]

FRP energy

\[
\int_{V_R} \psi_R(\sigma_R) \, dV = \frac{\chi}{\chi} \left[ \frac{1}{2E_R} \int_{A_R} \sigma_R^2 \, dA d\theta \right]
\]
**Complementary energy**

\[
\Psi = \int_{\theta_i}^{\theta_f} \left[ \frac{1}{2E_M} H(\hat{c} - Q) \otimes H(\hat{c} - Q) \cdot J + \frac{\sigma_y^2 A_p}{2E_M} + \frac{T^2}{2G_M A_s} \right] R \, d\theta
\]

\[
Q = \sigma_y \begin{Bmatrix} A_p \\ S_p \end{Bmatrix}, \quad J = \tilde{J} + \hat{J}, \quad \tilde{J} = \begin{bmatrix} A_e & S_e \\ S_e & I_e \end{bmatrix}, \quad \hat{J} = \frac{E_R}{E_M} \begin{bmatrix} A_R & S_R \\ S_R & I_R \end{bmatrix}, \quad H = J^{-1}
\]

plastic stress resultants  
elastic part of the masonry  
FRP (homogenized)

The solution of the problem is determined minimizing the complementary energy under the equilibrium constraint.
$c_p$, equilibrated stress resultants due to the load $p$

$c_q$, equilibrated stress resultants due to the load $q$

$c_i$, self-equilibrated stress resultants due to the unknown action $x_i$
\[
\min \{ \Psi(\mathbf{c}) / \mathbf{c} \text{ equilibrated} \} \\
\mathbf{c} = \mathbf{c}_p + \lambda \mathbf{c}_q + \sum_{i=1}^{h} x_i \mathbf{c}_i = \begin{cases} 
\hat{\mathbf{c}}_p + \lambda \hat{\mathbf{c}}_q + \sum_{i=1}^{h} x_i \hat{\mathbf{c}}_i \\
T_p + \lambda T_q + \sum_{i=1}^{h} x_i T_i 
\end{cases} = \begin{cases} 
N_p + \lambda N_q + \sum_{i=1}^{h} x_i N_i \\
M_p + \lambda M_q + \sum_{i=1}^{h} x_i M_i \\
T_p + \lambda T_q + \sum_{i=1}^{h} x_i T_i 
\end{cases}
\]

Stationary condition of \( \Psi \)

\[
0 = \frac{\partial \Psi}{\partial x_j} = \int_{\theta_i}^{\theta_f} \frac{1}{E_M} H \left( \left( \hat{\mathbf{c}}_p + \lambda \hat{\mathbf{c}}_q + \sum_{i=1}^{h} x_i \hat{\mathbf{c}}_i \right) - \mathbf{Q} \right) \cdot \mathbf{\hat{c}}_j + \frac{T_j \left( T_p + \lambda T_q + \sum_{i=1}^{h} x_i T_i \right)}{G_M A_S} R d\theta
\]

\[
\mathbf{s}_q = \int_{\theta_i}^{\theta_f} \left( \frac{1}{E_M} H \mathbf{\hat{c}}_q \cdot \mathbf{\hat{c}}_j + \frac{T_j T_q}{G_M A_S} \right) R d\theta
\]

\[
\mathbf{s}_p = \int_{\theta_i}^{\theta_f} \left( \frac{1}{E_M} H \mathbf{\hat{c}}_p \cdot \mathbf{\hat{c}}_j + \frac{T_j T_i}{G_M A_S} \right) R d\theta
\]

\[
\mathbf{s}_Q = \int_{\theta_i}^{\theta_f} \frac{1}{E_M} \mathbf{Q} \cdot \mathbf{\hat{c}}_j R d\theta
\]

\[
\mathbf{s}_p + \lambda \mathbf{s}_q + \sum_{i=1}^{h} x_i \mathbf{s}_i - \mathbf{s}_Q = \mathbf{0}
\]

Compatibility equation
Arc-length technique

Compatibility equation

\[ s = s_p(A_e, A_p) + (\lambda_n + \Delta \lambda) s_q(A_e, A_p) + \sum_{i=1}^{h} (x_{i,n} + \Delta x_i) s_i(A_e, A_p) - s_Q(A_e, A_p) = 0 \]

Constraint equation

\[ \tilde{\chi}^2 - \Delta l^2 = 0 \]

Iterative procedure

\[ s = s^k + \sum_{i=1}^{h} \frac{\partial s}{\partial x_i} \bigg|_{s=s^k} \delta x_i + \frac{\partial s}{\partial \lambda} \bigg|_{s=s^k} \delta \lambda = s^k + K^k \delta x + S^k \delta \lambda = 0 \]

\[ \delta x = -\left( K^k \right)^{-1} s^k - \left( K^k \right)^{-1} S^k \delta \lambda^k = \delta \bar{x}^k + \delta x^k \]

\[ \Delta \lambda^{k+1} = \Delta \lambda^k + \delta \lambda \]

\[ \Delta x^{k+1} = \Delta x^k + \delta x = \Delta x^k + \delta \bar{x}^k + \delta \lambda \delta x^k \]

\[ \delta \lambda^2 a_1 + \delta \lambda a_2 + a_3 = 0 \]
Iterative procedure

\[ 0 = \sum_{j=1}^{nt} \int_{\theta_j}^{\theta_{j+1}} \frac{1}{E_M} H^j \left[ \left( \hat{e}_p + \lambda \hat{e}_q + \sum_{i=1}^{h} x_i \hat{e}_i \right) - Q^j \right] \cdot \hat{c}_j + \frac{T_j \left( T_p + \lambda T_q + \sum_{i=1}^{h} x_i T_i \right)}{G_M A_S^j} R d\theta \]

1. Initialize the values of \( A_e, S_e, I_e, A_s, A_p, S_p \) in a finite number of the arch sections;
2. Solve the elastic equilibrium problem by minimizing the complementary energy under the equilibrium constraint for a value of the multiplier \( \lambda \);
3. Compute the stress resultants;
4. Evaluate the strain parameters (axial strain, bending curvature and shear deformation);
5. Determine the positions of the neutral and plasticity axes;
6. When the new section geometry \( A_e, S_e, I_e, A_s, A_p, S_p \) are computed, the residue is valued and by the arc-length technique a new value of the multiplier \( \lambda \) is determined;
7. If the residual is greater than a fixed tolerance then it is necessary to go back to point 2, otherwise the iterative procedure terminates.
Displacement formulation (FEM)

Timoshenko beam finite element displacement formulation 3-node element

- axial displacement quadratic interpolation,
- transverse displacement cubic interpolation, (quadratic enriched with a cubic bubble)
- rotation quadratic interpolation.

\[
\begin{align*}
\phi_1 &= \frac{1}{2} \xi(\xi - 1) \\
\phi_2 &= \frac{1}{2} \xi(\xi + 1) \\
\phi_3 &= 1 - \xi^2 \\
\psi &= \xi(\xi^2 - 1)
\end{align*}
\]

Implemented into FEAP (R.L. Taylor)
Post-computation of the shear stress

\[
\sigma_T = \frac{2N_R^-}{b(2R-h)} \\
\tau_T^- = \frac{2}{b(2R-h)} \frac{dN_R^-}{d\theta} \\
\sigma_T^+ = -\frac{2N_R^+}{b(2R+h)} \\
\tau_T^+ = -\frac{2}{b(2R+h)} \frac{dN_R^+}{d\theta}
\]
Heterogeneity of masonry

Local effects

\[ \tau = \tau_T + \tau_h \]
Repetitive cell

Micromechanical analysis

\[ \tau_h = K_{\tau} s_{\tau} \]

\[ s = GF \]
Numerical results

TEST 1

Geometry
- Masonry
  - $R=5000$ mm
  - Rectangular cross section: $b=300$ mm, $h=1000$ mm
- FRP
  - $t=0.17$ mm
  - $b=200$ mm

Materials properties
- Masonry
  - $E_m=15000$ MPa
  - $n=0.2$
  - $G_m=6250$ MPa
  - $\sigma_y=7.5$ MPa
- FRP
  - $E_{FRP}=400000$ MPa

- Round clamped arch;
- vertical downward distributed load;
- horizontal distributed load amplified by the multiplier $\lambda$;
- reinforcement at the extrados.
Unreinforced arch

- **STRESS APPROACH:**
  \( n_T = 300 \)

- **FINITE ELEMENT METHOD:**
  mesh of 60 elements

- NT, no tension material with unlimited compressive strength;
- NTP, no tension material with limited compressive strength;
- EC, complementary energy approach;
- FEM, elastoplastic displacement finite element formulation.
Curva delle pressioni
Sezioni reagenti
Collapse loading for the unreinforced and reinforced arch

![Graph showing collapse loading for the unreinforced and reinforced arch. The graph compares the effects of different FRP layer counts on the collapse loading. The x-axis represents the vertical displacement ($v_k$ [mm]), and the y-axis represents the load factor ($\lambda$). The graph includes curves for 1 FRP layer, 2 FRP layers, 5 FRP layers, 10 FRP layers, and 15 FRP layers. The unreinforced case is also shown for comparison. Different markers indicate compressive masonry failure and tensile FRP failure.]
Experimental validation [Briccoli Bati S., Rovero L. (2000)]

Geometry

Masonry
- $R=865$ mm
- $b=100$ mm
- $h=100$ mm
- Springs at $30^\circ$ and $150^\circ$

FRP
- $t=0.17$ mm
- $b=50$ mm

Materials properties

Masonry
- $E_{\text{brick}}=1785$ MPa
- $E_{\text{mortar}}=133$ MPa
- $E_m=680$ MPa
- $\nu=0.2$
- $G_m=283$ MPa

FRP
- $E_{\text{FRP}}=230000$ MPa

- flat arch;
- force $F$ at key;
- reinforcement at the intrados
Comparison between numerical and experimental results

- $f_y = 2.04$ MPa
- $f_y = 2.72$ MPa
- $f_y = 3.40$ MPa
- $f_y = 6.80$ MPa

Experimental results
Tangential shear stresses

Shear stress [MPa] vs. z* [mm]

Decohesion tangential stresses

Experimental failure
Arch

Geometry:
• $R_b=516$ mm
• $h=12$ mm $b=5.5$ mm

Mechanical properties:
• $E=8300$ N/mm$^2$
• $G=3400$ N/mm$^2$

FRP

Geometry:
• $t_f=0.17$ mm
• $b_f=10$ mm

Mechanical properties:
• $E=230000$ N/mm$^2$

Cancelliere-Sacco, 2007-2008
- Instrumentation
- Tests for the material characterization
- Tests of the arches without and with the reinforcement
Tests for the material characterization

Brick / Mixed mortar
a) compression test (UNI 8942/3)
b) indirect tensile test (UNI 8942/3)
c) secant elastic modulus (UNI 6556)

\[ f_{kk} = 14.9 \text{ N/mm}^2 \]
\[ f_{vk} = 1.79 \text{ N/mm}^2 \]

\[ R_f = 2.5 \text{ N/mm}^2 \]
\[ R_c = 8.80 \text{ N/mm}^2 \]
\[ E_s = 3100 \text{ N/mm}^2 \]
\[ E_b = 16000 \text{ N/mm}^2 \]
Tests on the arches

◆ **Tested specimens**

n. 2 specimens

◆ **Preparation of the specimens**

Construction  Unreinf. arch  FRP application  Reinf. arch
Arch 1
Arch 2
Response of the unreinforced arches

F(v) in corrispondenza del carico esterno
Arch 2 reinforced
Arch 2 reinforced
Damage of the reinforced arch
Response of the reinforced arch

F(v) in corrispondenza del carico esterno

Arco 2 R.
Remarks

- Different collapse mechanisms
- More than significant increase of the collapse load for the r.a.
- Collapse occurs for higher displacement values for the r.a.
Comparison between the experimental and numerical results

- n.r.a.

◆ n.r. arches (good agreement)

  - limit load
  - collapse mechanisms
r.a.

r. arch (good agreement)

- limit load
- failure
Conclusions

- The analysis could be considered quite reliable under certain limit:
  - The maximum compressive strain have to be checked
  - The FRP stress have to be checked
  - The FRP-masonry interaction could induce the FRP delamination.

- Although many simplifications have been introduced in the (constitutive model), the developed procedure (stress formulation) is quite complex.

- Although the model is simple, numerical results are in good agreement with the experimental evidences, in terms of collapse mechanisms and collapse load.

- ....
Developments

Coupled micro-macro analysis

Multiscale analysis

Masonry: heterogeneous material
Multiscale analysis

\[ u_G \]  Global displacements

\[ u_L \]  Local displacements (disturbances)

\[ u = u_G + u_L \]  Total displacements

Total potential energy

\[ \Pi = \Pi^G + \Pi^L + \Pi^{GL} \]

Stationary condition

\[
\begin{bmatrix}
K_{GG} & K_{GL} \\
K_{LG} & K_{LL}
\end{bmatrix}
\begin{bmatrix}
u_G \\
u_L
\end{bmatrix}
=\begin{bmatrix}
F_G \\
F_L
\end{bmatrix}
\]

A multiscale analysis

\[ K_{GG} u_G^k + K_{GL} u_L^{k-1} = F_G \]

compute \( u_G^k \)

Jacoby iterative resolution of the system of equations

\[ K_{LG} u_G^k + K_{LL} u_L^k = F_L \]

compute \( u_L^k \)

Other approach: domain decomposition
Thanks for your attention